

Cvičenia & analýzy

15b → 3x5b písomky (cca od 4.8.11. týždňa)
→ domáce úlohy (ak distancuje)

Robíme to, čo sme robili už na 3 predmetoch.

- nie vždy: $x > 2$ 3 uľahčujúca forma
- dvo uľahč: $\forall x \in \mathbb{R}: x > 2$
- negácia $\forall$ je $\exists$

9/11 a) $\forall x \in \mathbb{R}: x^2 \geq x$
 $\exists x \in \mathbb{R}: x^2 < x$ (pravdivý)

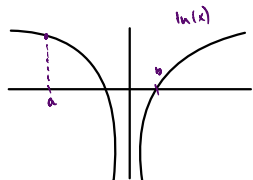
c) $\forall x \in \mathbb{R} \forall y \in \mathbb{R}: x > y \Rightarrow x^2 > y^2$
 $\exists x \in \mathbb{R} \exists y \in \mathbb{R}: x > y \wedge x^2 \leq y^2$ (pravdivý)

b) $\exists x \in \mathbb{R} \forall y \in \mathbb{N}: y \neq x$ (pravdivý) d) $\forall x, y \in \mathbb{R} \exists z \in \mathbb{N}: x < z \wedge y < z$
 $\forall x \in \mathbb{R} \exists y \in \mathbb{N}: y = x$

e) Nech a, b sú nejaké reálne čísla, potom z nerovnosti $\ln(a) > \ln(b)$ vyplýva $a > b$.

$\forall a, b \in \mathbb{R} \setminus \{0\}: \ln(a) > \ln(b) \Rightarrow a > b$

$\exists a, b \in \mathbb{R} \setminus \{0\}: \ln(a) > \ln(b) \wedge a \leq b$ (pravdivý)



DŮKAZY

právy ($A \Rightarrow B: A \Rightarrow a_1, a_2 \Rightarrow a_3 \dots \Rightarrow a_n \Rightarrow B$)
neprávy
dôkaz sporom
neprávy ($\neg A \Rightarrow B$)

dôkaz ekvivalencie $A \Leftrightarrow B \rightarrow$ treba dokázať obe implikácie ($\Rightarrow, \Leftarrow$)

9/2

metóda a postulujúce podmienky

1. Aby sme si 2 celých čísel bol deliteľný druhá, stačí aby každý z nich bol deliteľný druhá.
2. Aby celé číslo bolo deliteľné 100, je nutné aby bolo deliteľné 10.
3. Aby platila nerovnosť $\frac{1}{x} < 1$, stačí aby $x > 1$.
4. Aby platilo $\frac{1}{x} < 1$, je nutné a stačí aby bolo $x > 1$ alebo $x < 0$. ← TO TOTO ZNAMENÁ!

MNOŽINY

$\mathbb{R}$ - reálne

$\mathbb{R} \setminus \mathbb{Q}$ - iracionálne

$\mathbb{N}$ - prirodzené

$\mathbb{Z}$ - celé

$\mathbb{Q}$ - racionálne (zlomky)

$\mathbb{C}$ - komplexné

10/3

$\forall a \in \mathbb{Q}, b \in \mathbb{R} \setminus \mathbb{Q}$ (iracionálne): $a + b \in \mathbb{R}$

$a \in \mathbb{Q}, b \in \mathbb{R} \setminus \mathbb{Q} \Rightarrow a + b \in \mathbb{R}$
 $a + b = \frac{a}{1} + \frac{b}{1} = \frac{a+b}{1} \Rightarrow b = \frac{c}{a} \Rightarrow b \in \mathbb{Q}$
 $\frac{a}{1} + b = \frac{a+b}{1}$
 $\frac{a}{2} + b = \frac{a+b}{2}$ neplatí

$\mathbb{Q} + \mathbb{Q} = \mathbb{Q}$

$\mathbb{Q} + \mathbb{R} \setminus \mathbb{Q} = \mathbb{R}$

$\mathbb{R} \setminus \mathbb{Q} + \mathbb{R} \setminus \mathbb{Q} = \mathbb{R} \quad (\pi, -\pi = 0)$

10/4 $a, b \in \mathbb{R}, \sqrt{ab} \in \mathbb{R} \setminus \mathbb{Q} \Rightarrow \sqrt{a} + \sqrt{b} \in \mathbb{R} \setminus \mathbb{Q}$

nech platí
 a súčasne $\sqrt{a} + \sqrt{b} \in \mathbb{Q}$
 $\mathbb{Q} \rightarrow (\sqrt{a} + \sqrt{b})^2 = \frac{a}{1} + 2\sqrt{ab} + \frac{b}{1}$
 $\downarrow$
 $\mathbb{Q} \Rightarrow \text{spor (j to } \mathbb{R} \setminus \mathbb{Q})$

10/5 a) $\sqrt{5} \in \mathbb{R} \setminus \mathbb{Q} \Rightarrow \text{platí}$

sporom: $\sqrt{5} \in \mathbb{Q}$
 $\sqrt{5} = \frac{p}{q} \mid^*$
 $5 = \frac{p^2}{q^2} \mid^*$
 $5q^2 = p^2$ $\left. \begin{array}{l} \text{deli} \\ (5 \mid p^2 \Rightarrow 5 \mid p) \end{array} \right\}$
 $a = 5k_1^2$
 $5b^2 = (5k_1^2)^2$
 $b^2 = 5k_1^2$
 $b = 5k_2^2$
 $\sqrt{5} = \frac{5k_1}{5k_2} \rightarrow \frac{k_1}{k_2} \Rightarrow \text{neplatí}$

b) $\sqrt{5} \in \mathbb{R} \setminus \mathbb{Q}$
 spor: $\sqrt{5} \in \mathbb{Q}$
 $\sqrt{5} = \frac{p}{q} \mid^*$
 $5 = \frac{p^2}{q^2}$
 $5q^2 = p^2$

c) $\sqrt{2} + \sqrt{3} \in \mathbb{R} \setminus \mathbb{Q}$
 $2, 3 \in \mathbb{Q}$
 $\sqrt{2}, \sqrt{3} \in \mathbb{R} \setminus \mathbb{Q}$
 $\Rightarrow \sqrt{2} + \sqrt{3} \in \mathbb{R} \setminus \mathbb{Q}$
 $\downarrow$
 $2 + 2\sqrt{6} + 3 = \frac{p}{q}$
 $\sqrt{6} = \frac{p}{q}$

$\rightarrow$ ak máme
 hrdenu a, kt.
 sme si už
 dokázali, že platia, stačí to len zosumiť

D. U. doplniť

10/6 a) $\forall a, b \in \mathbb{R}^+, \frac{1}{2} \ln a + \frac{1}{2} \ln b \leq \ln \frac{a+b}{2}$

$\ln \sqrt{a} + \ln \sqrt{b} = \ln \sqrt{ab}$
 $\Rightarrow \ln \sqrt{ab} \leq \ln \frac{a+b}{2} \mid e^*$ $\left(\begin{array}{l} \text{vieme to odstrániť, lebo je to rastúca funkcia} \\ \text{(monotónne sme e na to)} \end{array} \right)$ **(NECHAPEM)**
 $\sqrt{ab} \leq \frac{a+b}{2}$
 $2\sqrt{ab} \leq a+b$
 $0 \leq a+b-2\sqrt{ab}$
 $0 \leq a-2\sqrt{ab}+b$
 $0 \leq (\sqrt{a}-\sqrt{b})^2$

$\left(\begin{array}{l} \text{ak by sme mali stať, tak} \\ \text{to vieme spraviť} \end{array} \right)$

b) $a \in \mathbb{R}, b > 0; c > 0; a < b \Rightarrow \frac{a}{b} < \frac{a+c}{b+c} \approx 0 < \frac{a+c}{b+c} - \frac{a}{b} = \frac{b(a+c) - a(b+c)}{b(b+c)} = \frac{ba+bc-ab-ac}{b(b+c)} = \frac{c(b-a)}{b(b+c)} \Rightarrow$
 $\frac{a}{b} < \frac{a+c}{b+c}$
 $\frac{c}{b+c} > 0$
 $\frac{b-a}{b+c} > 0$

$\hookrightarrow$ Keď máme pozitívne aj kladné čísla

10.2021 $(a+b)(a^2-ab+b^2)$

6. a) $(a+b) \geq 0 \Rightarrow a^3+b^3 \geq a^2b+ab^2$
 $a^3+b^3-a^2b-ab^2 \geq 0 \rightarrow \text{toto chceme ukázať}$
 $a^3-a^2b+b^3-ab^2 \geq 0$
 $a^2(a-b)+b^2(b-a) \geq 0$
 $a^2(a-b)+b^2 \cdot (-1)(a-b) \geq 0$
 $(a^2-b^2)(a-b) \geq 0$
 $(a-b)(a+b) \cdot (a-b) \geq 0$
 $(a-b)^2(a+b) \geq 0$
 $\hookrightarrow$ toto vieme $\hookrightarrow$ toto je zadané

d) $a^2+b^2+c^2 \geq ab+bc+ac$
 $a^2+b^2+c^2-ab-bc-ac \geq 0$
 $(a-b)^2 + (a-c)^2 + (b-c)^2 \geq 0$
 $a^2-2ab+b^2+a^2-2ac+c^2+b^2-2bc+c^2 \geq 0$
 $2(a^2+b^2+c^2) \geq 2(ab+bc+ac) \mid :2$
 $a^2+b^2+c^2 \geq ab+bc+ac$

$$e) \forall a, b \in \mathbb{R}^+ : \frac{a^2+b^2}{2} \geq \left(\frac{a+b}{2}\right)^2$$

$$\frac{a^2+b^2}{2} - \left(\frac{a+b}{2}\right)^2 \geq 0$$

$$\frac{a^2+b^2}{2} - \frac{a^2+2ab+b^2}{4} \geq 0$$

$$\frac{4a^2+4b^2-a^2-2ab-b^2}{4} \geq 0$$

$$\frac{3a^2-2ab+3b^2}{4} \geq 0$$

$$\frac{3}{4} (a^2-2ab+b^2) \geq 0$$

↳ toto sme uvideli
predtým už
dokázali

$$f) \frac{x^2+2}{\sqrt{x^2+1}} \geq 2$$

$$x^2+2 \geq 2\sqrt{x^2+1}$$

$$x^2+2-2\sqrt{x^2+1} \geq 0$$

$$x^2+1+1-2\sqrt{x^2+1} \geq 0$$

$$(\sqrt{x^2+1}-1)^2 \geq 0$$

MATEMATICKÁ INDUKCIA

$$\forall n \in \mathbb{N} : P(n)$$

- postup: 1) ukážeme, že výrok platí pre $n=1$ (najmenšie n)
2) predpokladáme, že platí pre n členov (indukčný predpoklad)
3) a IP ukážeme, že platí pre $n+1$

11/8

$$a) \forall n \in \mathbb{N} :$$

$$1^2+3^2+\dots+(2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

$$1) n=1$$

$$1^2 = \frac{1}{3} \cdot 1 \cdot (2 \cdot 1) \cdot (2 \cdot 1 + 1)$$

$$1 = 1 \quad \checkmark$$

$$II) IP: 1^2+3^2+\dots+(2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

III) dokme užitím IP

$$1^2+3^2+\dots+(2n-1)^2+(2n+1)^2 = \frac{1}{3}n(2n-1)(2n+1) + (2n+1)^2$$

FAKTA STANOVIA:

$$= \frac{1}{3}n(2n-1)(2n+1) + (2n+1)^2 =$$

$$= (2n+1) \left[\frac{1}{3}n(2n-1) + (2n+1) \right] =$$

$$= (2n+1) \left[\frac{n(2n-1) + 6n+3}{3} \right] =$$

$$= (2n+1) \frac{1}{3} (n(2n-1) + 6n+3) =$$

$$= (2n+1) \frac{1}{3} (2n^2+5n+3) =$$

$$= \frac{1}{3} (n+1)(2n+1)(2n+3)$$

LS = PS

$$b) \forall n \in \mathbb{N} : 1^2-2^2+3^2-4^2+\dots+(-1)^{n-1}n^2 = (-1)^{n-1} \cdot \frac{1}{2}n(n+1)$$

$$1) n=1$$

$$1^2 = (-1)^0 \cdot \frac{1}{2} \cdot 1 \cdot 2$$

$$1 = 1 \quad \checkmark$$

$$II) IP: 1^2-2^2+3^2-4^2+\dots+(-1)^{n-1}n^2 = (-1)^{n-1} \cdot \frac{1}{2}n(n+1)$$

III) z IP uk, že platí pre $n+1$

$$1^2-2^2+\dots+(-1)^{n-1}n^2+(-1)^n(n+1)^2 = (-1)^{n-1} \cdot \frac{1}{2}n(n+1) + (-1)^n(n+1)^2$$

$$CS: 1^2-2^2+\dots+(-1)^{n-1}n^2+(-1)^n(n+1)^2 =$$

$$(-1)^{n-1} \cdot \frac{1}{2}n(n+1) + (-1)^n(n+1)^2 =$$

$$= (-1)^n \left[-\frac{1}{2}n(n+1) + (n+1)^2 \right] =$$

$$= (-1)^n(n+1) \left(-\frac{1}{2}n + n+1 \right) =$$

$$= (-1)^n(n+1) \left(\frac{-n+2n+2}{2} \right) =$$

$$= (-1)^n(n+1) \left(\frac{n+2}{2} \right) = (-1)^n(n+1) \frac{1}{2}(n+2) \quad \checkmark \quad \underline{LS=PS}$$

$$d) \forall n \in \mathbb{N} : \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} > \frac{13}{24}$$

$$1) n=2$$

$$\frac{1}{3} + \frac{1}{4} > \frac{13}{24}$$

$$\frac{4}{12} + \frac{3}{12} > \frac{13}{24} \quad \checkmark$$

$$II) IP: \frac{1}{n+1} + \dots + \frac{1}{2n} > \frac{13}{24}$$

$$III) \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n} + \frac{1}{2n+2} > \frac{13}{24} \quad (z IP)$$

$$CS: \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n} + \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{n+1}$$

$$> \frac{13}{24} + \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{n+1}$$

muss ta byť ≥ 0

inak by sme to nepuknulo toto

$$> \frac{13}{24} + \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{n+1}$$

$$= \frac{13}{24} + \frac{2n+2+2n+1-(4n+2)}{(2n+1)(2n+2)}$$

$$= \frac{13}{24} + \frac{2n+3-4n-2}{(2n+1)(2n+2)}$$

$$= \frac{13}{24} + \frac{1}{(2n+1)(2n+2)} > 0$$

↳ toto je
určite kladné

$$> \frac{13}{24} \quad \checkmark$$

c) $\forall n \in \mathbb{N} : 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}$

1) $n=2$

$1 + \frac{1}{\sqrt{2}} > \sqrt{2} \checkmark$

2) $n=3$: $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} > \sqrt{3}$

3) $n \rightarrow n+1$: $1 + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} > \sqrt{n+1}$

$1 + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} > \sqrt{n} + \frac{1}{\sqrt{n+1}}$

$\frac{\sqrt{n} \cdot \sqrt{n+1} + 1}{\sqrt{n+1}} > \sqrt{n+1} \checkmark$

$\frac{n+1}{\sqrt{n+1}} = \sqrt{n+1}$

BUDUĆI

TRŽDENÍ

ROZCVIČKA

(2-3 příklady) dělat str. 10 (ex. 6) i se 2 stran
lábile dělat indukci

+ specifické m.k. D.V.



8.10.2024

M/10

a) $x > -1, n \in \mathbb{N}$
 $(1+x)^n \geq 1+nx$

1) $n=1$

$1+x \geq 1+x \checkmark$

$n=2$

$(1+x)^2 \geq 1+2x$

$1+2x+x^2 \geq 1+2x \checkmark$

2) IP: $(1+x)^n \geq 1+nx$

3) $n+1$: $(1+x)^{n+1} \geq 1+(n+1)x$

$(1+x)^n (1+x) \geq (1+nx)(1+x)$

IP $\parallel$

$1+x+nx+nx^2 = 1+x(1+n)+nx^2 \geq 1+x(n+1)$

b) $n \in \mathbb{N}; x_1, \dots, x_n \in \mathbb{R}$, mají rovnou znaménko
 > -1

$(1+x_1)(1+x_2) \dots (1+x_n) \geq 1+x_1+x_2+\dots+x_n$

1) $n=1$

$(1+x_1) \geq 1+x_1 \checkmark$

$n=2$

$(1+x_1)(1+x_2) \geq 1+x_1+x_2$

$1+x_1+x_2+x_1x_2 \geq 1+x_1+x_2 \checkmark$

2) IP: $(1+x_1)(1+x_2) \dots (1+x_n) \geq 1+x_1+\dots+x_n$

3) $(1+x_1)(1+x_2) \dots (1+x_n)(1+x_{n+1}) \geq 1+x_1+\dots+x_n+x_{n+1}$

$(1+x_1)(1+x_2) \dots (1+x_n) \geq 1+x_1+\dots+x_n$

$\underbrace{1+x_1+\dots+x_n}_{\geq 1+x_1+\dots+x_n} (1+x_{n+1}) \geq 1+x_1+\dots+x_n+x_{n+1}$

?

celkom první písaný příklad

M/11

a) $\forall n \in \mathbb{N}, n > 1$
 $n! < 2^n$

1) $n=2$: $2 < 2^2 \checkmark$

IP: $n+1 < 2^{n+1}$

1: $(n+1)! < 2^{n+1}$

$n+2 < 2^{n+1}$

stačí dok.: $\frac{n+2}{n+1} \leq \frac{2^{n+1}}{2^n}$

$1 + \frac{1}{n+1} \leq 2 \checkmark$

$\frac{n+1}{n+1} + \frac{1}{n+1}$

$\frac{n+1}{n+1} + \frac{1}{n+1}$

$\forall n \in \mathbb{N}$
 $a_n < b_n$
 $a_{n+1} < b_{n+1}$
stačí dokázat, že

$\frac{a_{n+1}}{a_n} \leq \frac{b_{n+1}}{b_n}$

$0 < a_n < b_n$
 $0 < \frac{a_{n+1}}{a_n} \leq \frac{b_{n+1}}{b_n} \Rightarrow a_{n+1} < b_{n+1}$

$1 < 2$
 $1 < 2$
specifický případ
nechť a_n
aroma b_n
 $\rightarrow 1 \leq 1$

$0 < A < B$
 $0 < R \leq S$
 $0 < AR < BS$

c) $\forall n \in \mathbb{N}$
 $(2n)! < 2^{2n} \cdot (n!)^2$ ✓

$n=1$
 $2! < 2^2 \cdot 1^2$
 $2 < 4$ ✓

$2, 1$ IP: $(2n)! < 2^{2n} \cdot (n!)^2$
 $n! \cdot (2n)! < 2^{2(n+1)} \cdot ((n+1)!)^2$
 $(2n+2)! < 2^{2n+2} \cdot ((n+1)!)^2$

staci: $\frac{(2n+2)!}{(2n)!} \leq \frac{2^{2n+2} \cdot ((n+1)!)^2}{2^{2n} \cdot (n!)^2}$
 $\frac{(2n+2) \cdot (2n+1) \cdot (2n)!}{(2n)!} \leq \frac{2^{2n+2} \cdot ((n+1)!)^2}{2^{2n} \cdot (n!)^2} = \frac{2^2 \cdot (n+1)! \cdot (n+1)!}{(n!) \cdot (n!)}$
 $(2n+2)(2n+1) \leq 2^2 \cdot (n+1)(n+1) = 2^2 \cdot \frac{(n+1)!}{n!} \cdot \frac{(n+1)!}{n!}$
 $4n^2 + 2n + 4n + 2 \leq 4n^2 + 4n + 4$
 $2n + 2 \leq 4n + 4$
 $n + 1 \leq 2n + 2$ ✓

c) $\forall n \in \mathbb{N}, n \geq 2$
 $2! \cdot 4! \cdot 6! \dots (2n)! > [(n+1)!]^n$

$n=2$
 $2! \cdot 4! > (3!)^2$
 $2 \cdot 24 > 6^2$
 $48 > 36$ ✓

$2, 1$ IP: $2! \cdot 4! \cdot 6! \dots (2n)! > [(n+1)!]^n$
 $n! \cdot 2! \cdot 4! \dots (2n)! > [(n+2)!]^{n+1}$
 $\frac{2! \cdot 4! \cdot (2n)!}{2! \cdot 4! \cdot (2n)!} \geq \frac{[(n+2)!]^{n+1}}{[(n+1)!]^n} = \frac{[(n+2)!]^{n+1}}{[(n+1)!]^n} \cdot \frac{(n+2)!}{(n+1)!}$

$(2n+2)! \geq (n+2)(n+2)!$
 $(n+2)! \cdot (n+4) \dots (2n+2) \geq (n+2)^n \cdot (n+2)!$
 $(n+2) \cdot (n+4) \dots (2n+2) \geq (n+2)^n$
 $n \text{ členů}$
 $\text{každý} > n+2$

d) $\forall n \in \mathbb{N}, n > 1$
 $n! < \left(\frac{n+1}{2}\right)^n$

$n=2$
 $2! < \left(\frac{3}{2}\right)^2$
 $2 < \frac{9}{4}$
 $\frac{8}{4} < \frac{9}{4}$ ✓

$2, 1$ IP: $n! < \left(\frac{n+1}{2}\right)^n$
 $n! \cdot (n+1)! < \left(\frac{n+2}{2}\right)^{n+1}$

staci dok.: $\frac{(n+1)!}{n!} < \frac{\left(\frac{n+2}{2}\right)^{n+1}}{\left(\frac{n+1}{2}\right)^n}$

$\frac{(n+1)n!}{n!} \leq \frac{\left(\frac{n+2}{2}\right)^n \cdot \left(\frac{n+2}{2}\right)}{\left(\frac{n+1}{2}\right)^n}$

$(n+1) \leq \frac{\left(\frac{n+2}{2}\right)^n \cdot \frac{n+2}{2}}{\left(\frac{n+1}{2}\right)^n}$

$n+1 \leq \frac{(n+2)^n \cdot (n+2)}{2 \cdot (n+1)^n} \quad / \cdot 2, : (n+1)$

$2 \geq \left(\frac{n+2}{n+1}\right)^{n+1}$

$\left(\frac{n+2}{n+1}\right)^{n+1} = \left(1 + \frac{1}{n+1}\right)^{n+1} \geq 1 + (n+1) \cdot \frac{1}{n+1} = 2$

f) $\forall n \in \mathbb{N}, n \geq 3$
 $n^{n+1} > (n+1)^n$

$n=3$
 $3^4 > 4^3$
 $81 > 64$ ✓

$2, 1$ IP: $n^{n+1} > (n+1)^n$
 $n! \cdot (n+1)^{n+1} > (n+2)^{n+1}$

staci dok.: $\frac{(n+1)^{n+2}}{n^{n+1}} \geq \frac{(n+2)^{n+1}}{(n+1)^n}$

$\frac{(n+1)^n \cdot (n+1)^2}{n^{n+1}} \geq \frac{(n+2)^n \cdot (n+2)}{(n+1)^n}$

$(n+1)^n \cdot (n+1)^2 \geq n^{n+1} \cdot (n+2)^n \cdot (n+2)$

$(n+1)^{2n+2} \geq (n(n+2))^{n+1}$

$(n+1)^{2n+2} \geq (n^2 + 2n)^{n+1}$

$(n+1)^{2(n+1)} \geq (n^2 + 2n)^{n+1}$

$(n+2n)^{n+1} \geq (n^2 + 2n)^{n+1}$ ✓

11/9

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$a) |x| = \max \{x, -x\}$$

$$\begin{array}{ll} \text{I. } x \geq 0 & \text{II. } x < 0 \\ |x| = x & |x| = -x \\ x \geq -x & -x \geq x \end{array}$$

NEDOPISANÉ POZNÁMKY

20. DŮA 15.10.2021

22. 10. 2021

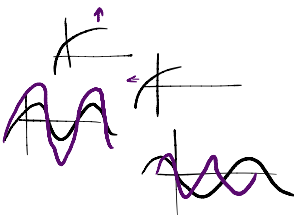
GRAFY

$y = f(x) + c$ ← posun o c v směru y

$y = f(x, c)$ ← posun v směru x

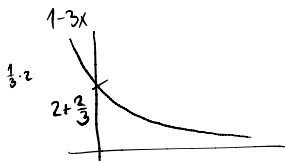
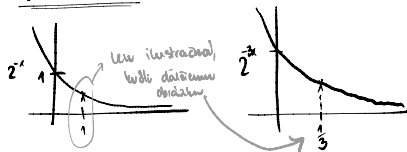
$y = a \cdot f(x)$ ← násobení y -ové souřadnice

$y = f(ax)$ ← násobení x -ové souřadnice



19/27.12

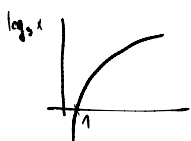
$$y = \frac{1}{3} 2^{1-3x} + 2$$



$$\begin{aligned} 2^{1-3x} & x=0 \Rightarrow F=2 \\ 2^{-3x} & x=0 \Rightarrow F=1 \end{aligned}$$

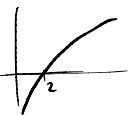
19/27.15

$$y = \log_3(0.5x + 2)$$



$$3^0 = 1$$

$$\log_3(0.5x)$$



$$3^0 = 0.5x$$

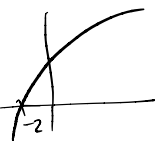
$$1 = 0.5x$$

$$2 = x$$

$$\log_3(0.5x + 2)$$

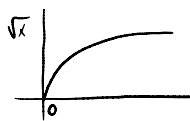
$$1 = 0.5x + 2$$

$$-2 = x$$

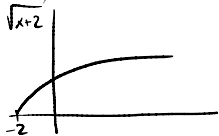


19/27.9

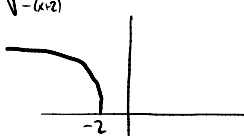
$$y = -\sqrt{-x-2}$$



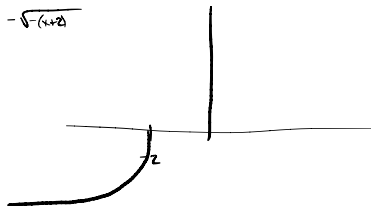
$$D(f) = (-2, \infty)$$



$$D(f) = (-\infty, -2]$$



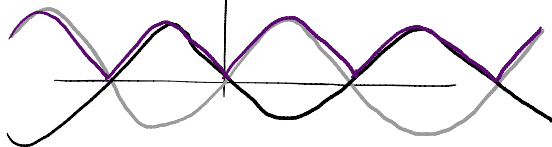
$$-\sqrt{-x-2}$$



$$\sin |x|$$

$$\sin x$$

$$|\sin x|$$



$$\sin \frac{1}{x}$$

$$\sin x$$

$$x > 1 \Rightarrow \frac{1}{x} < 1$$

$$x < 1 \Rightarrow \frac{1}{x} > 1$$

nekonečne
veľa
periód

v strednom fíziku
↳ pískajú (10 bodov)
(búd na ciele
alebo v strede
večer)



↳ budú tam veľa 2 ciele

$$x \cdot \sin x$$

$$f(x) = \log_x 10$$

$$f^{-1}(x)$$

1. f - presta?

2. f⁻¹?

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

$$\log_{x_1} 10 = \log_{x_2} 10$$

$$\log_{10} 10 = \log_{x_1} 10$$

$$\log_{x_1} 10 = \log_{x_2} 10$$

$$x_1 = x_2$$

f je prostá

$$y = \log_x 10$$

$$x = \log_y 10$$

$$x = \frac{\log 10}{\log y}$$

$$\log_y 10 = \frac{1}{x}$$

$$10^{\frac{1}{x}} = y$$

$$D(f^{-1}) = \mathbb{R} \setminus 0$$

↳ ale aby som
mohol povedať, že je
to funkciou potrebujem
aj jej $D(f^{-1})$, nie len
predpis

24/53.4

$$\frac{2x}{1+x^2}, x \in (-1, 1)$$

21 $f^{-1}: x = \frac{2y}{1+y^2}$

$$\frac{2x_1}{1+x_1^2} = \frac{2x_2}{1+x_2^2} \quad | \cdot \frac{1}{2}$$

$$x_1(1+x_2^2) = x_2(1+x_1^2) = 0$$

$$x_1 + x_2^2 x_1 - x_2 - x_2 x_1^2 = 0$$

$$x_2 x_1 (x_1 - x_2) + (x_1 - x_2) = 0$$

$$(x_1 - x_2) \cdot (1 - x_1 x_2) = 0$$

1) $x_1 = x_2$

2) $1 - x_1 x_2 = 0$

$$1 = x_1 x_2$$

↳ nastane pr. $x_2 \neq x_1$ } f je
postla

$$x + x y^2 = 2y$$

$$y^2 x - 2y + x = 0$$

$$y_{1,2} = \frac{2 \pm \sqrt{4 - 4x^2}}{2x}$$

$$y_{1,2} = \frac{1}{x} \pm \frac{\sqrt{1-x^2}}{x}$$

$$D(f^{-1}) = (-1, 1) \setminus \{0\}$$

↳ toto sú ale 2 funkcie
a funkcia má byť
len 1 inverzná!

$$f^{-1} = \frac{1 - \sqrt{1-x^2}}{x}$$

potrebujeme, aby
mala hodnoty
v $(-1, 1)$

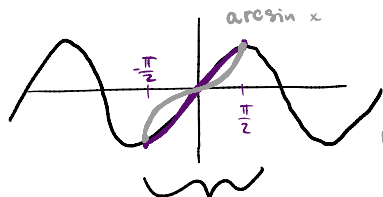
$$\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}$$

↳ ak dosadíme
veľké číslo, tak
budeme mať veľké
číslo, ale tie
menšie po seitaní
v intervale $(-1, 1)$

chcem ich
odškrtnúť

25/59.1

$$f(x) = \sin^3 x, x \in (-\frac{\pi}{2}, \frac{\pi}{2})$$



inverzná f
má len tento
úsek, nie celý $\sin x$

$$f = \sin x, D(f) = (-\frac{\pi}{2}, \frac{\pi}{2}), H(f) = (-1, 1)$$

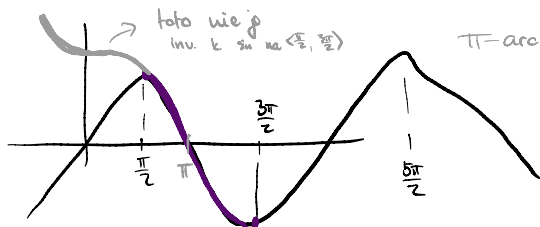
$$f^{-1} = \arcsin x, D(f^{-1}) = (-1, 1), H(f^{-1}) = (-\frac{\pi}{2}, \frac{\pi}{2})$$

$$x = \sin^3 y$$

$$\sqrt[3]{x} = \sin y \quad / \arcsin$$

$$\arcsin \sqrt[3]{x} = y$$

$$f(x) = \sin^3 x, (-\frac{\pi}{2}, \frac{\pi}{2})$$



$$\pi - \arcsin \sqrt[3]{x}$$

25/52.1

$$\arcsin\left(\sin \frac{\pi}{3}\right)$$

$$\arcsin(\sin x) = x$$

toto platí len keď
 $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\sin \frac{\pi}{3} = \sin \frac{2\pi}{3}$$

odpovedť môžeme
 pomocou o kosoťuholníku
 najjednoduchšie periódy

$$\arcsin\left(\sin \frac{\pi}{3}\right) = \frac{\pi}{3}$$

$\sin(\arccos x) \rightarrow$ čo sa spoznáť
 pomocou
 Pytagorovej
 vetky

$\sin \alpha = ?$
 $\alpha = \arccos x \quad | \cdot \cos$
 $\cos \alpha = x$

Pytagorova veta

$\sin \alpha = \frac{\sqrt{1-x^2}}{1}$
 $\sin(\arccos x) = \sqrt{1-x^2}$

KÚL

25/60.2

$$\sin(2 \arcsin x) \rightarrow \text{použijeme vzorec}$$

$$= 2 \frac{\sin(\arcsin x)}{1} \cos(\arcsin x)$$

$$= 2x \cdot \sqrt{1-x^2}$$

$$\begin{aligned} \tan(2 \arctan x) & \rightarrow \text{to si} \\ & \downarrow \frac{\sin 2x}{\cos 2x} = \frac{\sin(x+2x)}{\cos(x+2x)} \\ & \text{alebo} \\ & \text{nie,} \\ & x-ka \end{aligned}$$

$$\cos(\arcsin x)$$

$$\cos \alpha = ?$$

$$\arcsin x = \alpha \quad | \cdot \sin$$

$$x = \sin \alpha$$

$$\cos(\arcsin x) = \sqrt{1-x^2}$$

26.6.2021

PROSTÁ FUNKCIA

↳ stačí, aby bola rydžomonotónna
 (rastúca / klesajúca)

$f \circ g$ - rydžomon. $\Leftrightarrow$ f - rydžomon.
 $\wedge g$ - rydžomon.

5.6.2021

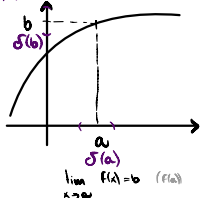
ROZCVIKY NA

CVIKACH (limity) r. Wagowanie na kusoach

cvičenia: 116, 117, 120, 121

LIMITY

a) funkcie



ak vezmeme body z dedia bodu a ,
 tak všetky funkčné hodnoty
 budú patriť do dia bodu b

limita $\neq$ limita
 funkcie postupnosti

$$\forall \delta(b) \exists \delta(a) \forall x \in (\delta(a) - \delta(a)) \cap D(f) : f(x) \in \delta(b)$$

$$\forall \varepsilon > 0 \exists \delta > 0 : |x - a| < \delta \Rightarrow |f(x) - b| < \varepsilon$$

$$x \in D(f)$$

b) posthypoteti

$$\lim_{n \rightarrow \infty} a_n = b$$

$$\forall \varepsilon > 0 \exists n_0 \in \mathbb{N} : n > n_0 : |a_n - b| < \varepsilon$$

33/105.5 2 definice užitá:

$$\lim_{n \rightarrow \infty} q^n = 0, |q| < 1$$

$$|q^n - 0| < \varepsilon$$

$$|q^n| < \varepsilon \xrightarrow{\log_{1/|q|}} |q| < 1 \text{ takže } \text{sa odíča zväčšovanie}$$

$$n \cdot 1 > \log_{1/|q|} \varepsilon$$

* ak by som použila ln

$$n \ln |q| < \ln \varepsilon$$

$$n > \frac{\ln \varepsilon}{\ln |q|}$$

$$n_0 = \lceil \frac{\ln \varepsilon}{\ln |q|} \rceil$$

lim f(x)

1. skúšam dosadiť -> limit to vyjde

$$\frac{\infty}{\infty} / \frac{0}{0} \rightarrow \text{musím upraviť}$$

$$0 \cdot \infty = -\infty \vee$$

$$\infty - \infty = ?$$

≥ veľký o limitách

$$\lim f \pm g =$$

$$= \lim f + \lim g$$

$$\lim f \cdot g =$$

$$= \lim f \cdot \lim g$$

$$\lim \frac{f}{g} =$$

$$= \frac{\lim f}{\lim g} \neq 0$$

38/116.1

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{2x + 1} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x} - \frac{1}{x}}{\frac{2x}{x} + \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{x - \frac{1}{x}}{2 + \frac{1}{x}} = \frac{1 - 0}{2 - 0 + 0} = \frac{1}{2}$$

x na najväčšiu mocninu

Funckije iba pre x → ∞

38/116.5

$$\lim_{x \rightarrow \infty} \frac{(x-1)(x-2) \dots (x-5)}{(5x-1)^5} = \lim_{x \rightarrow \infty} \frac{x-1}{5x-1} \cdot \lim_{x \rightarrow \infty} \frac{x-2}{5x-1} \cdot \lim_{x \rightarrow \infty} \frac{x-3}{5x-1} \cdot \lim_{x \rightarrow \infty} \frac{x-4}{5x-1} \cdot \lim_{x \rightarrow \infty} \frac{x-5}{5x-1} =$$

$$= \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x}}{5 - \frac{1}{x}} = \frac{1 - 0}{5 - 0} = \frac{1}{5}$$

38/116.6

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 5^n} = \frac{\frac{1}{3}}{\frac{1}{5}} = \lim_{n \rightarrow \infty} \frac{(-\frac{2}{3})^n + 1}{(-\frac{2}{5})^n + 3} = \frac{0 + 1}{0 + 2 + 3} = \frac{1}{5}$$

$$\lim_{n \rightarrow \infty} q^n = 0$$

38/117.2

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{2x^2 - x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)(2x+1)} = \frac{2}{3}$$

polynom / všimnime si (ak máme základy, ktoré musíme upraviť na základe)

38/117.3

$$\lim_{x \rightarrow 0} \frac{x^2 - 1}{2x - 1} = -1 \neq \text{dosadiť}$$

38/117.3

$$\lim_{n \rightarrow \infty} \frac{x^n - 1}{x^n - 1} = \frac{(x-1)(x^{n-1} + \dots + 1)}{(x-1)(x^{n-1} + \dots + 1)} = \frac{x}{x}$$

polymoring
ak to vyjasa
a nie g to bava
vyjimam
konca
metoda to polnu
(metoda to polnu na 2)

delenie polynomov

$$(x^n - 1) : (x - 1) = x^{n-1} + x^{n-2} + x^{n-3} + \dots + x + 1$$

$$\frac{x^n - 1}{x - 1}$$

$$\frac{x^{n-1} + x^{n-2} + \dots + x + 1}{1}$$

39/120.1

$$\lim_{x \rightarrow 2} \frac{\sqrt{x-6} + 2}{\sqrt{x-6} - 1} = \lim_{x \rightarrow 2} \frac{x-6 + 4}{(\sqrt{x-6} + 2)(\sqrt{x-6} - 1)} = \lim_{x \rightarrow 2} \frac{x-2}{(\sqrt{x-6} + 2)(\sqrt{x-6} - 1)}$$

už som len dosadil $-2 = \frac{2}{-24}$

$$A^2 - B^2 = (A+B)(A-B)$$

$$\frac{A^2 - B^2}{A+B} = A-B$$

$$A^2 - B^2 = (A+B)(A-B)$$

$$\frac{A^2 - B^2}{(A+B)(A-B)} = \frac{A-B}{A+B}$$

ak je to 3 odmocnina
može byť pod ňou aj
záporné číslo, čo
má veľa záporné

↑ toto pomáha s
odmocninami v limitách

39/120.3

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x} - \sqrt{x-1}}{\sqrt{x} - x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x} - x} = 0 + \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x} - \sqrt{x-1}}{\sqrt{x} - x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x} - x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x} - x} = \frac{0}{\infty} = 0$$

39/120.5

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - \sqrt{x-1}}{x + 2\sqrt{x+1}} = \lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{x+1}}}{x + 2\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

40/121.1

$$\lim_{x \rightarrow 2} \frac{\sqrt{x-2} - \sqrt{x+1}}{x-2} = \lim_{x \rightarrow 2} \frac{\frac{1}{2\sqrt{x-2}}}{x-2} = \lim_{x \rightarrow 2} \frac{1}{2\sqrt{x-2}(x-2)} = \frac{1}{4} - \frac{1}{3} = -\frac{1}{12}$$

ak to bude
má hodnota 2

a) $\lim_{x \rightarrow 2} \frac{\sqrt{x-2}}{x-2} = \lim_{x \rightarrow 2} \frac{1}{\sqrt{x-2}} = \frac{1}{\sqrt{2-2}} = \frac{1}{0}$

b) $\lim_{x \rightarrow 2} \frac{2 - \sqrt{x^2-4}}{x-2} = \lim_{x \rightarrow 2} \frac{2 - \sqrt{(x-2)(x+2)}}{x-2} = \lim_{x \rightarrow 2} \frac{2 - \sqrt{(x-2)(x+2)}}{(x-2)(\sqrt{x+2} + 2)} = \frac{2}{4+2\sqrt{2}} = \frac{1}{2+\sqrt{2}}$

